SEPTEMBER 5, 2017.

$\boxed{\text{LAST CLASS}}$

regular operations: ① union $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$

② concatenation $A \circ B = \{xy \mid x \in A \text{ and } y \in B\}$

③ star $A^* = \{x_1 \dots x_k \mid k \geq 0 \text{ and } x_i \in A \ \ \forall 1 \leq i \leq k\}$

1) closure. – Does applying a function to a regular language output a new regular language

Q: Are the regular languages closed under union?

(DFA input ⇒ 1)

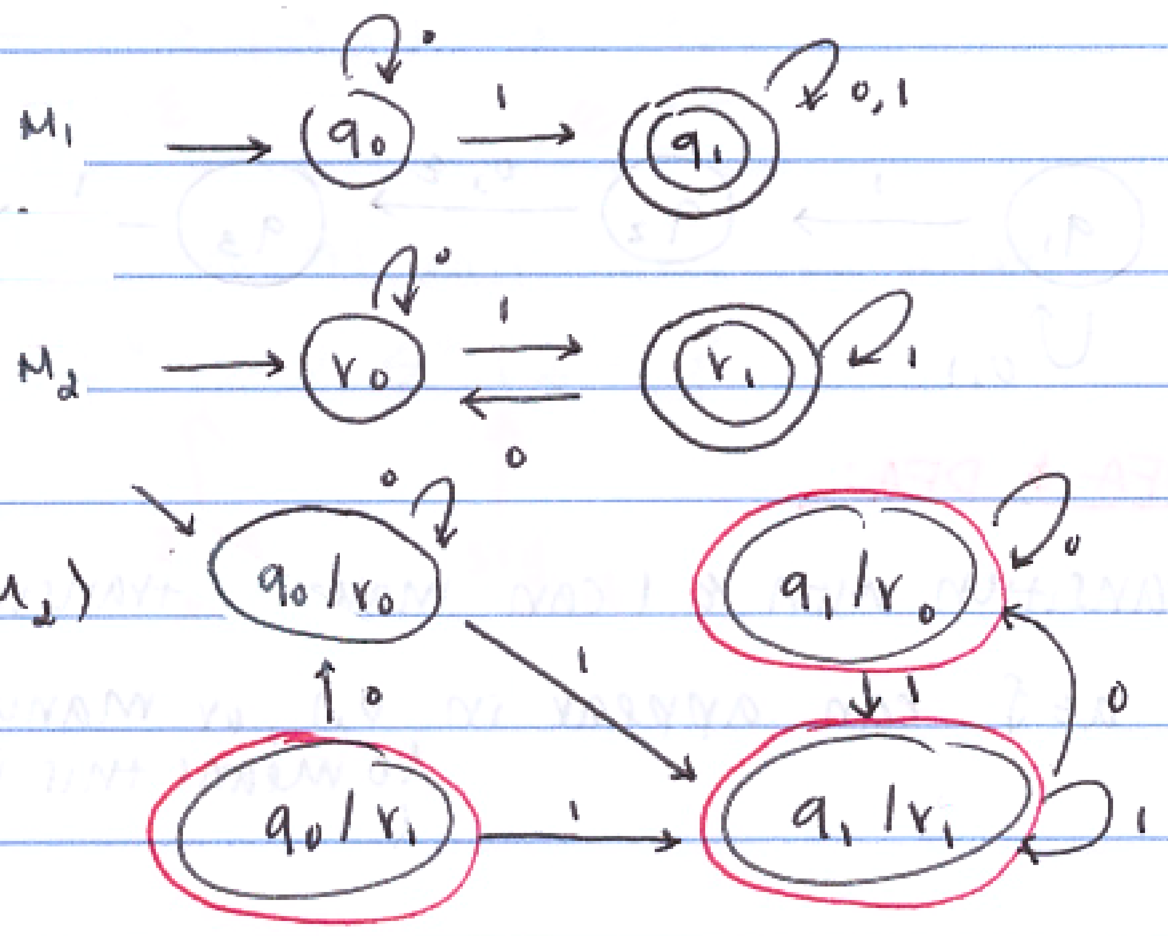
i.e. given regular A & B, is $A \cup B$ regular?

regular

A = YES!

PROOF SKETCH:

ex. to inspire us.



M_3 recognize $L(M_1) \cup L(M_2)$

Idea: let Q_1 & Q_2 be the sets of states for M_1 and M_2, respectively

① Define the state space for the M_3 as $Q_1 \times Q_2$

② Simultaneously track transitions for M_1 & M_2

③ Accept if end in state (q_i, r_j) s.t. $q_i \in Q_1$ or $r_j \in Q_2$ is accepting for M_1 or M_2, respectively.

FORMAL PROOF: let DFA $M_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ recognize language A_1

let $M_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$ recognize language A_2

we construct the DFA $M_3 = (Q_3, \Sigma, \delta_3, q_3, F_3)$ s.t. $L(M_3) = A_1 \cup A_2$

① $Q_3 = Q_1 \times Q_2 = \{(r_1, r_2) \mid r_1 \in Q_1, r_2 \in Q_2\}$

② Σ unchanged

current state, symbol read, M_1, M_2

③ For all $(r_1, r_2) \in Q_3$, and all $a \in \Sigma$ let $\delta((r_1, r_2), a) = (\delta_1(r_1, a), \delta_2(r_2, a))$

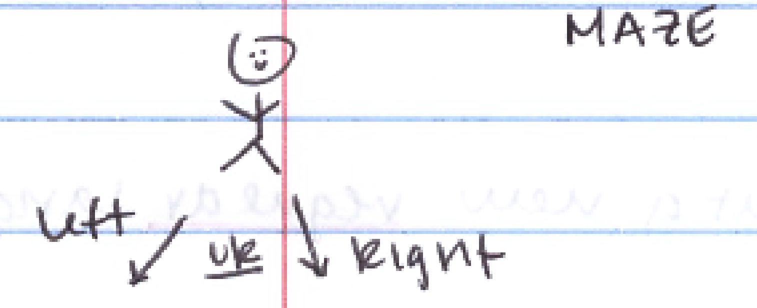
④ $q_3 = (q_1, q_2)$

⑤ $F_3 = \{(r_1, r_2) \mid r_1 \in F_1 \text{ or } r_2 \in F_2\}$ ∎

∴ Regular languages closed under union.

So far: Deterministic FA.

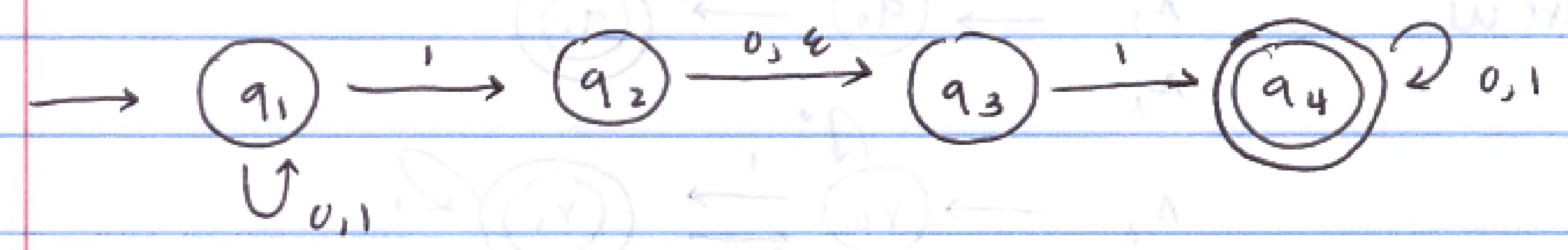
NONDETERMINISTIC FA (NFA)



NFA = "DFA where at each step it can follow multiple choices simultaneously"

$L(N_1) = \{x \mid$ x contains 11 or 101 as substring $\}$

NFA N_1

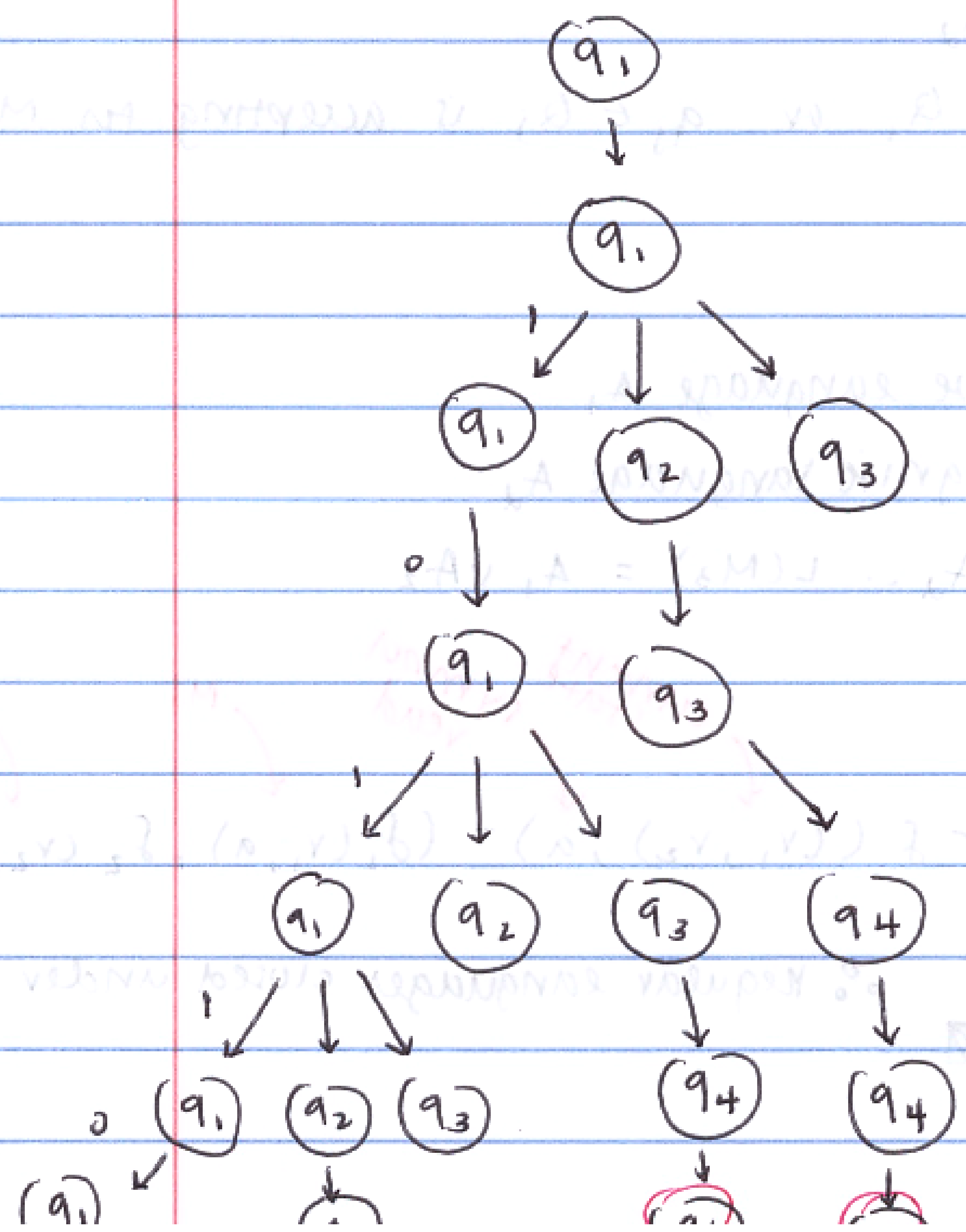


Differences b/t NFA & DFA:

① can label transition with ε (can make a transition without reading input)

② each symbol $a \in \Sigma$ can appear in 0, 1, or many transitions (0 means this branch of computation stops)

let's draw computation tree for N_1 on input $x = 010110$

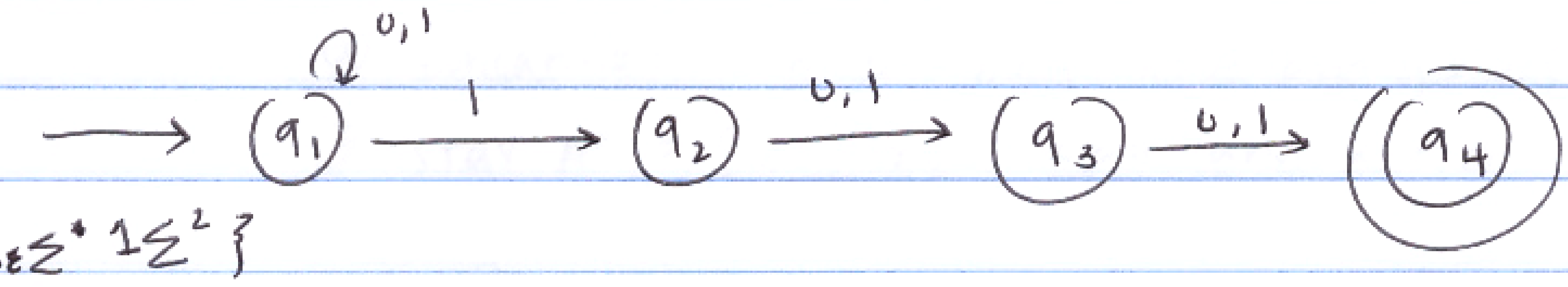


"ACCEPT": if there exists computational branch which leads to an accept state.

∴ N_1 accepts input x

Designing NFAs

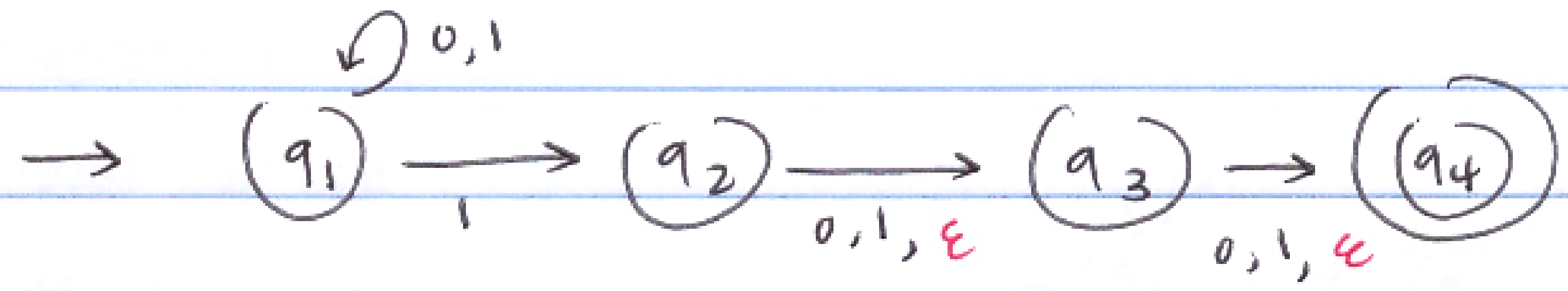
e.g. N_2



$L(N_2) = \{ x \mid x \in \Sigma^* 1 \Sigma^2 \}$

(the third-last symbol is 1)

e.g. N_2'



$L(N_2') = \{ x \mid x$ has 1 in one of its last three positions

$x \in \Sigma^* 1 \cup \Sigma^* 1 \Sigma \cup \Sigma^* 1 \Sigma^2 \}$

last symbol

2nd last

3rd last